Irreversible investments with private information and complementarities.

Abstract

Consider the case that investments are irreversible, and that different investors have different private information about the profitability of investments. Then, the proportion of investors that actually invests is smaller when there are complementarities and further decreases with the number of investors. In most circumstances, higher volatility decreases the proportion of investors that actually invest.

Keywords: uncertainty, coordination, complementarities, information.

YEL-codes: C72, D82, E22, F21.

1. Introduction

Irreversible investments under uncertainty can explain a wide range of phenomena. Dixit and Pindyck (1998) showed that the irreversibility of investments lead investors to demand a very high expected return on their investments before they make the irreversible investments. When the investments are also complementary, the hold-up problem may appear (e.g. De-Fraja, 1999 and Koss and Eaton, 1997). Hold-up problems have been identified in the labour market (e.g. Picot and Schlicht (eds.), 1997), in markets for environmental regulation (e.g. Gersbach and Glazer, 1999), and also for foreign direct investments and investments during agricultural transitions (Gow and Swinnen, 1998). In this paper I look at the hold-up problem when agents have private information about the future benefits of investments. This for instance arises when foreign investors who invest in complementary activities have private expectations about future exchange rates, or when workers and employees who both have to make match-specific investments have private information about future market circumstances, future personal circumstances or planned government interventions. When an employee for instance considers investing in match-specific capital, he is likely to have private information about his future health. The employer considering match-specific investments with that employee is likely to have private information about the value of future production. Similarly, different investors in an industry are likely to have some private information about the future developments in their particular sub-sector. These are examples of private information about the future profits of all investors in the complementary investments. The private information is obtainable to other agents in principle, but at a cost. Considering the role of private information in these circumstances seems worthwhile therefore.

In a simple model I consider the case that individuals have private knowledge about the profitability of all investments in a complementary activity. The main result is that complementarities increase the probability that a minimal proportion of the investors will believe the investment to be unprofitable, making the investment unprofitable for all. Hence greater uncertainty coupled with private information and complementarities yields a hold-up problem, which does not arise without private information.

In the second section, the model is presented and some links are made to the literature on complementarities. The final section concludes.

2. The model

2.1. The setting

Consider the decision of an agent when he has to make known costs now and when his future revenues are uncertain. More precisely, assume that an irreversible investment would cost Q now and would yield future revenues of Qε. Several interpretations of this set-up arise. Q could stand for the cost of physical capital or labour costs in the case that the investments are interpreted as being made by firms, in which case ε could represent future prices or exchange rates when it concerns investments abroad. When the agent is a worker who has the option of investing in match-specific capital, Q could be interpreted as the costs of this capital, whereas ε would then be a function of the probability that the match will last and would also depend on the future prices of the activities of the worker in case a worker obtains part of this future production. Another example would be when the agent is a firm who considers following environmental regulation, where Q would represent the costs of following the regulation and ε would represent an unknown benefit from an improved future environment or an unknown compensation by the authorities.

The actual profit of an agent i in case he invests equals $\pi_i = Q(\varepsilon - 1)$. Suppose that the uncertain factor ε is affected by n shocks, which are each i.i.d.:

$$\varepsilon = \gamma + \sum_{j=1}^{n} u_j \tag{2.1}$$

whereby γ is a constant and the cumulative distribution of u_j is denoted by G$(u_j; \mu, \frac{1}{n}\sigma^2)$ which is defined, continuous and differentiable for all $u_j \geq 0$. There holds that E$[u_j]$=$\mu = 0$, E$[(u_j - \mu)^2]$=$\frac{1}{n}\sigma^2$, $\frac{dG(u_j;\mu,\frac{1}{n}\sigma^2)}{d(\frac{1}{n}\sigma^2)} \geq 0$ iff $u_j < \mu$ and $0 \geq \frac{dG(u_j;\mu,\frac{1}{n}\sigma^2)}{d(\frac{1}{n}\sigma^2)}$ iff $u_j > \mu$. These last assumptions imply that there is less probability mass near μ when σ^2 increases (the more volatility, the less small shocks arise). It also implies that G must be unimodal with its median and mean coinciding. These assumptions are met by a wide range of distributions, including the normal distribution and the uniform distribution. For an individual with no information on any of these shocks, there holds E$[\varepsilon]$=γ, and Var$[\varepsilon]$=σ^2.

Now suppose that investor i has information on one of the random shocks before he decides to invest, i.e., investor i knows the realization of one of the u_j's which is denoted by u_i. The other random shocks are revealed after the decision to invest. Hence the investor i knows the realization of u_i but only knows the distribution of other shocks, which means that for investor i there holds that E$[\varepsilon_i]$=γ+u_i and Var$[\varepsilon_i]$= $(1-\frac{1}{n})\sigma^2$. We can interpret $\frac{1}{n}$ as a measure of the amount of information held by an investor: the greater the amount of information, the less uncertain the investor is about future profits. This private information may either be information that is available publicly, but which would require a costly search effort of other investors to find, or information that is genuinely not available to others, such as inside information about intended policies of governments or intended actions by other major organizations (union, institutional investors, etc.). In case the investor is an individual, it could also contain private information about personal circumstances, for instance the state of health of an individual employee.

In the absence of investment complementarities, the ex-ante probability that a risk-neutral investor i invests, which is denoted by P_i, equals the probability that his expected profits are positive[1], which is

$$\begin{aligned} P_i &= \Pr[E[Q(\varepsilon_i - 1)] > 0] && (2.2) \\ &= \Pr[Q(\gamma + u_i - 1)) > 0] \\ &= \Pr[u_i > 1 - \gamma] \\ &= \bar{G}(1 - \gamma; 0, \frac{1}{n}\sigma^2) && (2.3) \end{aligned}$$

[1] P_i denotes the ex ante probability that investor i will invest; the investor himself knows with certainty, knowing the realisation of u_i, whether he will invest. P_i can in this case be interpreted as the proportion of potential investors that actually invests.

with E the expectations operator and where Pr denotes probability. If the investment is expected to be profitable in the absence of information (i.e., $\gamma > 1$), then P_i decreases with the volatility of ε ($=\sigma^2$), and decreases in the amount of private information an investor i has ($=\frac{1}{n}$). This result is not new: in any investment model, if private information suggests an investment is unprofitable, an investor will not invest.

2.2. Extreme complementarities

Suppose now that there are k potential investors and that the pay-offs to the investments of the k investors are complementary and that the investors have to decide simultaneously whether to invest in a particular industry or not. For simplicity, the complementarities between the investments are extreme such that revenues are zero if one or more investors have not invested. Although there is quite some evidence of for the existence of complementarities (e.g. Bartelsman et al., 1994, Cooper and John, 1988, Cooper and Haltiwanger, 1996, and Miyagawa, 1993), this assumption is somewhat extreme. In section 2.3, it is therefore shown that a weaker form of complementarity leads to qualitatively similar results.

The complementarities mean that the expected profit of actually investing by investor i can be written as:

$$\hat{\pi}_i = Q(P_{\neq i}\hat{\varepsilon}_i - 1) \tag{2.4}$$

whereby $\hat{\varepsilon}_i$ denotes the expectation of i as to ε and $P_{\neq i}$ denotes the ex ante probability, as evaluated by i, that all the investors apart from i will invest. For simplicity only, it is assumed that each investor has a different piece of information, i.e. no two potential investors know the realization of the same u_j.[2]

The probability that the expected profit of investor i is positive equals

$$\begin{aligned} P_i &= \Pr(\hat{\pi}_i > 0) && (2.5) \\ &= \Pr[Q(P_{\neq i}(\gamma + u_i) - 1) > 0] \\ &= \bar{G}(\frac{1}{P_{\neq i}} - \gamma; 0, \frac{1}{n}\sigma^2) && (2.6) \end{aligned}$$

Now, if all agents share the same expectations of the investment behaviour of all other agents, the ex ante probability that any investor invests equals P_i. Furthermore, because each investor has a separate piece of information, the ex ante probability that all other investors apart from i invests is equal to $(P_i)^{k-1} = P_{\neq i}$.

[2] Investors who have the same information as another investor will behave identically in the equilibria we look at. Hence we need only examine the k investors with different pieces of information.

We can then solve for P_i as a nested function of P_i:

$$P_i = \bar{G}(\frac{1}{(P_i)^{k-1}} - \gamma; 0, \frac{1}{n}\sigma^2) = f(P_i) \tag{2.7}$$

A solution to this equation is a Nash equilibrium in the sense that the belief of each investor i about the ex ante probability that any other investor will invest, is equal to the ex ante probability that investor i will invest. One can also view this equilibrium in terms of reservation strategies: an individual will invest only if u_i is greater than $\frac{1}{(P_i)^{k-1}} - \gamma$. If we denote this value as the reservation value r_i of individual i and do the same for all others, $P_{\neq i}$ equals $\Pr[u_1 > r_1, .., u_{i-1} > r_{i-1}, u_{i+1} > r_{i+1}, ..u_k > r_k]$. If all individuals have correct beliefs about the reservation strategies of all others, they must have the same reservation values in this case. Solving for the common reservation value is then equivalent to solving for P_i. The latter is the route expanded upon here.

In general, the solution equation is not analytically tractable for every conceivable distribution function G, but its characteristics are tractable.

First, one equilibrium that always exists is $P_i = 0$ because $f(0) = \bar{G}(\infty) = 0$. If each investor believes no-one else will invest, he will with certainty also not invest. We also know that 1≥f(1). The maximum number of equilibria possible then equals 1+2s, where s is the number of maxima of $\frac{d\bar{G}(\frac{1}{(P_i)^{k-1}}-\gamma;0,\frac{1}{n}\sigma^2)}{dP_i}$. If $\frac{d\bar{G}(\frac{1}{(P_i)^{k-1}}-\gamma;0,\frac{1}{n}\sigma^2)}{dP_i}$ is unimodal (as with the normal distribution), the number of equilibria is at least one and at most three. Hence, as in Cooper and John (1988), it is possible to get multiple investment equilibria due to the coordination problem between investors in the presence of complementarities.

Another characteristic of (2.7) is that any solution corresponds to a $\bar{G}(1-\gamma) \geq P_i$. This inequality is strict if the marginal distribution $g(.)$ is positive and continuous at $(1-\gamma)$. Therefore, the probability that investments are expected to be profitable will generally be lower with complementarities than in the absence of complementarities and will never be higher. Also, when k goes to infinity, the only remaining solution is $P_i = 0$. Therefore, even if the effect of volatility on investment is negligible for the expected profits of one investor in the non-complementary case as he may have only very little private information, the effect becomes non-negligible when the investments of many investors are complementary.

As to the other characteristics of the equilibrium: by taking total derivatives, we obtain

$$\frac{dP_i}{d\gamma} = \frac{\frac{\partial f(P_i)}{\partial \gamma}}{1 - \frac{\partial f(P_i)}{\partial P_i}} = \frac{g(\frac{1}{(P_i)^{k-1}} - \gamma; 0, \frac{1}{n}\sigma^2)}{c}$$

$$\frac{dP_i}{dk} = \frac{\frac{\ln P_i}{(P_i)^{k-1}} g(\frac{1}{(P_i)^{k-1}} - \gamma; 0, \frac{1}{n}\sigma^2)}{c}$$

$$\frac{dP_i}{d\sigma^2} = \frac{\frac{\partial \bar{G}(\frac{1}{(P_i)^{k-1}} - \gamma; 0, \frac{1}{n}\sigma^2)}{\partial(\sigma^2)}}{c}$$

where $c = 1 - \frac{\partial f(P_i)}{\partial P_i} = 1 - \frac{(k-1)}{(P_i)^k} g(\frac{1}{(P_i)^{k-1}} - \gamma; 0, (1 - \frac{1}{n})\sigma^2)$. If $0 \geq c$, the equilibrium is unstable. Looking only at the stable equilibria, there holds that $\frac{dP_i}{d\gamma} \geq 0$ and $0 \geq \frac{dP_i}{dk}$. These inequalities are strict for distributions with positive marginal distributions for the whole real axis. Hence, the probability of investments will generally increase with γ and decrease with the number of investors. Also, $0 \geq \frac{dP_i}{d\sigma^2}$ iff $\gamma > \frac{1}{(P_i)^{k-1}}$. Hence, when investments are expected to be unprofitable in the absence of information ($\gamma < 1$), increasing volatility makes these investments more likely because the probability that all investors simultaneously have very positive private information increases. If investments are expected to be very profitable however, these investments are going to be less likely to occur when there is private information and volatility increases.

As an example of the possibilities, take G(.) to be a normal distribution

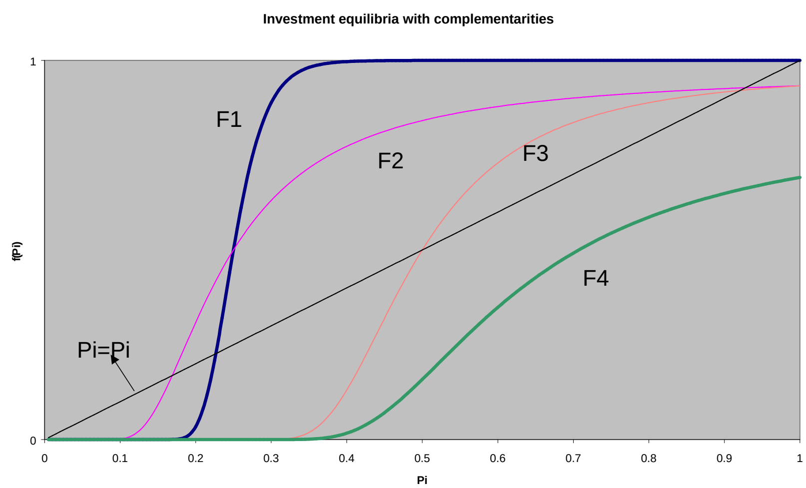

Figure 2.1: Parameter values for F1: n=10, k=2, $\gamma = 4, \sigma^2 = 3$. F2: n=10, k=2, $\gamma = 4, \sigma^2 = 40$. F3: n=10, k=3, $\gamma = 4, \sigma^2 = 40$. F4: n=10, k=3, $\gamma = 2, \sigma^2 = 40$.

and consider the four examples in Figure 1 where P_i is plotted against f(P_i) for various parameter values. Each intersection point is an equilibrium, but only the intersections at which $\frac{\partial f(P_i)}{\partial P_i} < 1$ are stable.

In the figure, each function has a stable intersection point at P$_i = 0$. F1, with very high γ, has two more intersection points: one very near P$_i = 1$ and one where P$_i \approx 0.228$. This last one is unstable. For F2, with a higher volatility (=σ^2), we also have three intersection points, and we can see that the highest intersection point is somewhat lower due to the increased volatility. For F3, where k is higher relative to F2, we still have three intersection points, but the highest intersection point is somewhat lower, due to the decreased γ. For F4, profitability is lower than for F3, which means P4 has only one equilibrium at $\mathrm{P}_i = 0$.

We may note that if no investor has any private information, there is only one Nash-equilibrium if $\gamma < 1$, which is that no-one invests. If iff $\gamma > 1$ and no-one has any information, there are two Nash-equilibria: one where no-one invests and one in which each investor will, in which case there is no hold-up problem, whatever the volatility of future profits.

2.3. generalized complementarities

So far, the complementarities were assumed to be extreme in the sense that the irreversible investments were a total write-off when one of the k investors did not invest. It seems likely however that when there are many investors, it is not so much important that every investor invests, but that a sizeable proportion invests. To incorporate this possibility, define future revenues as:

$$Q\epsilon * f(\frac{\sum_{i=1}^{k} y_i}{k})$$

where $y_i = 1$ iff investor i invests and $y_i = 0$ otherwise; As a minimum set of assumptions on the function f(.), it is assumed that $f(1) = 1$, $fs(0) = 0$, and $\frac{\partial fs}{\partial k} > 0$. Hence revenues for all increase when a higher proportion of potential investors invests. The amount of future revenue of investor i then equals $y_i Q\epsilon f(\frac{\sum y}{k})$. The probability that m out of k investors will actually invest equals $(P_i)^m(1-P_i)^{k-m}\binom{k}{m}$. What is important for the individual investor is the expected amount of sales should he himself decide to invest. This equals $Q\epsilon\overline{f}_i{=}Q\epsilon\sum_{m=0}^{m=k-1}\binom{k-1}{m}(P_i)^m(1-P_i)^{k-m-1}f(\frac{1+m}{k})$. There holds that $\frac{\partial\overline{f}_i}{\partial P_i}\geq 0$, and $0\geq\frac{\partial\overline{f}_i}{\partial k}$. The intuition behind this last result is that the importance of the information the investor has about his own decision to produce decreases with greater k: when calculating the profits he would make if he invests, investor i knows with certainty that a proportion of the investors will invest if he invests (namely himself). This proportion decreases with k. Hence the expected revenues of an investor are decreasing in k for any given $P_i < 1$.

For P_i there now holds:

$$
\begin{aligned}
P_i &= pr(\hat{\pi}_i > 0) \\
&= pr[Q(\overline{f}_i(\gamma+u_i)-1) > 0] \\
&= \bar{G}\Big(\frac{1}{\sum_{m=0}^{m=k-1}\binom{k-1}{m}(P_i)^m(1-P_i)^{k-m-1}f(\frac{1+m}{k})}-\gamma;0,\frac{1}{n}\sigma^2\Big)
\end{aligned}
$$

for which we can see again that $P_i < \bar{G}(1-\gamma)$, implying that the presence of complementarities reduces the probability of investment. Also, $0\geq\frac{dP_i}{d\gamma}$ and

$0 \geq \frac{dP_i}{dk}$ for stable equilibria, as in the case with extreme complementarities. As to the effect of volatility, we can again see that if $\gamma > \frac{1}{f_i}$, then $\frac{dP_i}{d\sigma^2} < 0$. The proportion of potential investors that hence invests ($=P_i$) in projects which are otherwise quite profitable, decreases with the amount of information that each investor has and decreases with volatility. When the investment is otherwise expected to be unprofitable, it is possible that increasing volatility actually increases the probability of investment (from exceedingly unlikely to very unlikely). The qualitative results are therefore the same as in the case of extreme complementarities.

2.4. Supporting evidence

Hold-up problems in the labour market in the presence of complementarities have been well-documented and have lead to a wide variety of models (see e.g. the survey in Teulings and Hartog, 1997).

For foreign investments, there is some useful evidence on the relation between investments and volatility: when an investor has to make fixed costs in one currency and obtains sales in another currency, the problem described above may become relevant when different investors in complementary activities have private expectations about exchange rates. The empirical studies of Bell and Campa (1997), Goldberg and Kolstad (1995) and Campa (1993) have shown that bilateral investments into the countries they investigated declined with exchange rate volatility, especially when investments were irreversible. In the model in this paper, this would also happen for investments with a high γ. There are alternative explanations of these finding though, such as risk-aversion (Goldberg and Kolstad, 1995) or heterogeneity in time-preferences (Neumann, 1995). Indeed, if the efficient market hypothesis is true, investors could not have any information about future exchange rates not already discounted in current exchange rates. The efficient market hypothesis directly contradicts Schumpeter's perception of markets as ways to continuously find new information though. Even so, the applicability of the model to foreign investments relies on a violation of the efficient market hypothesis.

3. Conclusions

This paper makes a single simple point. When the profits of an investment are uncertain and different investors have private information on the future profitability of the investment, the number of investors will decline if these investments are complementary. If investments are otherwise expected to be quite profitable, higher volatility of future revenues increases the fraction of potential investors that has private information making them believe that an investment is unprofitable. Hence, uncertainty with private information in the case of complementary investments leads to a hold-up problem greater than without private information, even in the absence of risk-aversion.

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